6. Autocorrelation
Overview

1. Independence

2. Modeling Autocorrelation

3. Temporal Autocorrelation Example

4. Spatial Autocorrelation Example
6.1 Independence
6.1 Independence. Model assumptions

- All linear models (including SEM) assume that errors are independent, i.e., uncorrelated.

- Correlation can arise from:
  - Poor experimental design (Hurlbert 1984)
  - Natural phenomenon (e.g., temp & latitude, etc.)
  - Subsampling the same replicate
  - Spatial non-independence (sites closer together in space are more alike)
  - Temporal non-independence (periods closer in time are more alike, repeatedly sampling the same replicate)
6.1 Independence. Two kinds of correlations

1. Correlations among predictors

2. Correlations among observations
6.1 Independence. Collinearity

- Correlated predictors inflate standard errors & influence significance tests
6.1 Independence. Diagnosing collinearity

- SPLOM
- Variance inflation factors
- Moran’s I
pairs(data)

# Also see:
library(psych)
pairs.panel(data)

library(GGally)
ggpairs(data)
6.1 Independence. Variance inflation factors

- The proportion of variance in a predictor explained by all other predictors in the model: $1/(1 - R^2_j)$

- VIF < 2 is ideal (Zuur et al. 2009)

```r
data = data.frame(
    y = rnorm(100, 0, 1),
    x1 = rnorm(100, 0, 1),
    x2 = rnorm(100, 0, 1)
)

data$x3 = data$x1 + runif(100, 0, 2)
pairs(data)
```
6.1 Independence. Variance inflation factors

```r
model = lm(y ~ x1 + x2 + x3, data)

vif(model)

  x1    x2    x3
3.220394 1.004194 3.216532

# Drop correlated terms
model2 = update(model, . ~ . -x3)

vif(model2)

  x1    x2
1.001204 1.001204
```
Estimates correlation based on distance matrix

```r
random.sp.data = data.frame(
  y = rnorm(100, 0, 1),
  lat = sample(seq(36, 38, 0.1), 100, replace = T),
  long = sample(seq(-72, -70, 0.1), 100, replace = T)
)

order.sp.data = data.frame(
  y = c(runif(20, 0, 1), runif(20, 2, 10), runif(20, 10, 20),
  runif(20, 20, 40), runif(20, 40, 80)),
  lat = seq(36, 38, 0.02)[-1],
  long = seq(-72, -70, 0.02)[-1]
)

# Get spatial distance matrix
random.sp.dist = sp::spDists(as.matrix(random.sp.data[, c("long",
  "lat")]), longlat = T)

order.sp.dist = sp::spDists(as.matrix(order.sp.data[, c("long",
  "lat")]), longlat = T)
```
6.1 Independence. Moran’s I

Moran.I(random.sp.data$y, random.sp.dist)

$observed
[1] -0.01325627

$expected
[1] -0.01010101

$sd
[1] 0.005655542

$p.value
[1] 0.5769092

Moran.I(order.sp.data$y, order.sp.dist)

$observed
[1] -0.4392363

$expected
[1] -0.01010101

$sd
[1] 0.008284921

$p.value
[1] 0
6.1 Independence. Correlogram

Estimates correlation with increasing lag (distance) among data points

```r
random.correlog = correlog(random.sp.data$lat, random.sp.data$long, 
  z = random.sp.data$y, increment = 20, resamp = 10, latlon = T)

order.correlog = correlog(order.sp.data$lat, order.sp.data$long, 
  z = order.sp.data$y, increment = 20, resamp = 10, latlon = T)
```

![Correlogram](image1.png) ![Correlogram](image2.png)
6.1 Independence. Treating collinearity

- Centering and scaling
- Variable selection
- PCA
- Model correlations / covariances
6.2 Modeling Autocorrelation
6.2 Modeling Autocorrelation. Options

1. Specify fixed correlation structure

2. Fit random effects
Correlations among sampling points follow a predetermined pattern.

<table>
<thead>
<tr>
<th>Compound Symmetry</th>
<th>CS</th>
<th>UN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unstructured</td>
<td>UN</td>
<td></td>
</tr>
<tr>
<td>First-Order Autoregressive</td>
<td>AR(1)</td>
<td></td>
</tr>
</tbody>
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<td></td>
</tr>
</tbody>
</table>

For Compound Symmetry (CS), the correlation matrix is:

\[
\begin{bmatrix}
\sigma^2 + \sigma_1 & \sigma_1 & \sigma_1 & \sigma_1 \\
\sigma_1 & \sigma^2 + \sigma_1 & \sigma_1 & \sigma_1 \\
\sigma_1 & \sigma_1 & \sigma^2 + \sigma_1 & \sigma_1 \\
\sigma_1 & \sigma_1 & \sigma_1 & \sigma^2 + \sigma_1 \\
\end{bmatrix}
\]

For Unstructured (UN), the correlation matrix is:

\[
\begin{bmatrix}
\sigma_1^2 & \sigma_{21} & \sigma_{31} & \sigma_{41} \\
\sigma_{21} & \sigma_2^2 & \sigma_{32} & \sigma_{42} \\
\sigma_{31} & \sigma_{32} & \sigma_3^2 & \sigma_{43} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_4^2 \\
\end{bmatrix}
\]

For First-Order Autoregressive (AR(1)), the correlation matrix is:

\[
\begin{bmatrix}
\sigma^2 & \rho & \rho^2 & \rho^3 \\
\rho & 1 & \rho & \rho^2 \\
\rho^2 & \rho & 1 & \rho \\
\rho^3 & \rho^2 & \rho & 1 \\
\end{bmatrix}
\]
6.2 Modeling Autocorrelation. *nlme*

**nlme**: Linear and Nonlinear Mixed Effects Models

```r
install.packages("nlme")
library(nlme)
```
6.2 Modeling Autocorrelation. \textit{nlme}

?corClasses

\texttt{corAR1}
autoregressive process of order 1.

\texttt{corARMA}
autoregressive moving average process, with arbitrary orders for the autoregressive and moving average components.

\texttt{corCAR1}
continuous autoregressive process (AR(1) process for a continuous time covariate).

\texttt{corCompSymm}
compound symmetry structure corresponding to a constant correlation.

\texttt{corExp}
exponential spatial correlation.

\texttt{corGaus}
Gaussian spatial correlation.

\texttt{corLin}
linear spatial correlation.

\texttt{corRatio}
Rational quadratics spatial correlation.

\texttt{corSpher}
spherical spatial correlation.

\texttt{corSymm}
general correlation matrix, with no additional structure.
Driscoll & Roberts (1997): Impact of burning on recovery of Walpole frog (*Geocrinia lutea*)

Measured frog calls for 3 years pre- and post-burn in 6 different paired drainages (burnt & unburnt)
How does the difference in frog calls between burnt and unburnt plots change through time?

→ Fit within block (drainage) correlation structure to account for temporal autocorrelation within blocks
driscoll = read.csv("driscoll.csv")

# Fit and test different correlation structure
driscoll.none = gls(CALLS ~ YEAR, data = driscoll,
                    na.action = na.omit)

driscoll.unstr = gls(CALLS ~ YEAR, data = driscoll,
                     na.action = na.omit,
                     correlation = corSymm(form = ~ 1 | DRAINAGE))

driscoll.cs = gls(CALLS ~ YEAR, data = driscoll,
                  na.action = na.omit,
                  correlation = corCompSymm(form = ~ 1 | DRAINAGE))

driscoll.ar1 = gls(CALLS ~ YEAR, data = driscoll,
                   na.action = na.omit,
                   correlation = corAR1(form = ~ 1 | DRAINAGE))
# Compare models

```r
anova(driscoll.none, driscoll.unstr, driscoll.cs, driscoll.ar1)
```

<table>
<thead>
<tr>
<th>Model</th>
<th>df</th>
<th>AIC</th>
<th>BIC</th>
<th>logLik</th>
<th>Test</th>
<th>L.Ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>driscoll.none</td>
<td>1</td>
<td>117.8460</td>
<td>119.9702</td>
<td>-55.9230</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>driscoll.unstr</td>
<td>2</td>
<td>110.5396</td>
<td>114.7879</td>
<td>-49.2698</td>
<td>1 vs 2</td>
<td>13.3064</td>
<td>0.0040</td>
</tr>
<tr>
<td>driscoll.cs</td>
<td>3</td>
<td>111.6621</td>
<td>114.4943</td>
<td>-51.8310</td>
<td>2 vs 3</td>
<td>5.1225</td>
<td>0.0772</td>
</tr>
<tr>
<td>driscoll.ar1</td>
<td>4</td>
<td>110.8742</td>
<td>113.7064</td>
<td>-51.4370</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

```

summary(driscoll.none)$tTable

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-10.656863</td>
<td>-2.0183</td>
<td>0.0618</td>
</tr>
<tr>
<td>YEAR</td>
<td>5.416667</td>
<td>2.2254</td>
<td>0.0418</td>
</tr>
</tbody>
</table>

summary(driscoll.ar1)$tTable

<table>
<thead>
<tr>
<th>Value</th>
<th>Std.Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-12.039670</td>
<td>-2.8224</td>
<td>0.0129</td>
</tr>
<tr>
<td>YEAR</td>
<td>5.696191</td>
<td>3.7060</td>
<td>0.0021</td>
</tr>
</tbody>
</table>
1. Specify correlation structure
2. Fit random effects
3. Both??

Correlation structure accounts for long-term temporal trend

Random structure accounts for random variation among time points

See comments: https://dynamicecology.wordpress.com/2015/11/04/is-it-a-fixed-or-random-effect/
6.3 Temporal Autocorrelation Example

6_Autocorrelation.R
Byrnes et al (2011) – Disturbance impacts on food web structure in kelp forests

Annual community surveys of kelp forests + remote sensing data on kelp cover
6.3 SEM Example. Kelp food webs
6.3 SEM Example. Increasing storms

**East Pacific Winter Storm Intensity**

- **Y-axis:** Winter cm NTR Wave Height
- **X-axis:** Years (1945 to 1995)

**North Pacific Winter Cyclone Frequency**

- **Y-axis:** Number of Winter Cyclones
- **X-axis:** Years (1945 to 1995)
6.3 SEM Example. Sampling 2000-2009
6.3 SEM Example. Meta-model

Food Web Diversity and Structure

↑

Kelp

↑

Wave

Disturbance
6.3 SEM Example. Meta-model

Food Web Diversity And Structure

Summer Kelp

Spring Kelp

Winter Wave Disturbance
6.3 SEM Example. Meta-model

Food Web Diversity And Structure

Summer

Kelp

Spring

Kelp

Winter Wave Disturbance

Wave*Kelp Interaction

Last Year’s Kelp
6.3 SEM Example. Full model

- **Reef cover**
- **Linkage density**
- **Richness**
- **Summer kelp cover**
- **Spring kelp cover**
- **Wave disturbance**
- **Wave x previous kelp**
- **Previous kelp cover**
6.3 SEM Example. Formatting the data

# Read Byrnes data
byrnes = read.csv("byrnes.csv")

# Only examine complete cases (remove NAs)
byrnes = byrnes[complete.cases(byrnes),]

# Log transform some variables (as in original analysis)
byrnes$prev.kelp = log(byrnes$prev.kelp + 1)
byrnes$spring.kelp = log(byrnes$spring.kelp + 1)
byrnes$summer.kelp = log(byrnes$summer.kelp + 1)
# Create model with correlation structure
byrnes.sem <- psem(

  spring.kelp <- gls(spring.kelp ~ wave * prev.kelp + reef.cover, 
    correlation = corCAR1(form = ~ YEAR | SITE / TRANSECT), 
    data = byrnes),

  kelp <- gls(summer.kelp ~ wave * prev.kelp + reef.cover + spring.kelp, 
    correlation = corCAR1(form = ~ YEAR | SITE / TRANSECT), 
    data = byrnes),

  richness <- gls(richness ~ summer.kelp + prev.kelp + reef.cover + 
    spring.kelp, 
    correlation = corCAR1(form = ~ YEAR | SITE / TRANSECT), 
    data = byrnes),

  linkdensity <- gls(linkdensity ~ richness + summer.kelp + prev.kelp + 
    reef.cover + spring.kelp, 
    correlation = corCAR1(form = ~ YEAR | SITE / TRANSECT), 
    data = byrnes)
)

)
6.3 SEM Example. Goodness-of-fit

Goodness-of-fit:

Global model: Fisher's C = 1.912 with P-value = 0.752 and on 4 degrees of freedom

Individual R-squared:
  Response R.squared
  spring.kelp  0.17
  summer.kelp  0.34
  richness     0.04
  linkdensity  0.37
6.3 SEM Example. Full model

- Wave disturbance
- Wave x previous kelp
- Previous kelp cover
- Summer kelp cover
- Spring kelp cover
- Richness
- Linkage density
- Reef cover

Relationships:
- Wave disturbance: -0.32
- Wave x previous kelp: 0.26
- Summer kelp cover: 0.23
- Spring kelp cover: 0.59
- Richness: 0.15
- Linkage density: 0.47
- Reef cover: 0.24
6.3 SEM Example. Inferences

- Wave action reduces spring kelp cover.
- Spring and summer kelp cover enhances richness, and therefore food web structure.
- Wave action reduces food web structure:
  - $-0.32 \times 0.59 \times 0.15 \times 0.47 = -0.01$
6.3 SEM Example. Your turn...

Re-fit the model addressing the hierarchical structure of the sampling (hint: random effect of transect within site) *and* the temporal autocorrelation!

Compare using AIC...
6.3 SEM Example. Your turn...

- Same inferences
- Indirect effect:

\[-0.23 \times 0.57 \times 0.16 \times 0.50 = -0.01\]
6.3 SEM Example. Your turn...

- AIC (temporal structure only) = 61.912
- AIC (temporal & hierarchical structure) = 79.028
- GLS model incorporating temporal lag for transects within site sufficient (ΔAIC > 2)
- Random variation after accounting for temporal lag not significant
- Same answer (estimates) anyways
6.4 Spatial Autocorrelation Example

6_Autocorrelation.R
Harrison & Grace (2007) - Drivers of forest productivity in California

Spatially explicit forest and environmental data
boreal.sem <- psem(

    lm(richness ~ freezing, data = boreal),

    lm(NDVI ~ richness + freezing + wet, data = boreal),

    freezing %~~% wet,

    data = boreal

)
6.4 SEM Example 2. “Independent”

Goodness-of-fit:

Global model: Fisher's $C = 2.422$ with P-value $= 0.298$ and on 2 degrees of freedom

Individual R-squared:
- Response R.squared
  - richness $= 0.01$
  - NDVI $= 0.77$
6.4 SEM Example 2. “Independent”

Structural Equation Model of boreal.sem

Call:
richness ~ freezing
NDVI ~ richness + freezing + wet
freezing ~~ wet

AIC      BIC
18.422   52.65

Tests of directed separation:

<table>
<thead>
<tr>
<th>Independ.Claim</th>
<th>Estimate</th>
<th>Std.Error</th>
<th>DF</th>
<th>Crit.Value</th>
<th>P.Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>richness ~ wet + ...</td>
<td>-35.13</td>
<td>33.7123</td>
<td>530</td>
<td>-1.0421</td>
<td>0.2979</td>
</tr>
</tbody>
</table>

Coefficients:

<table>
<thead>
<tr>
<th>Response</th>
<th>Predictor</th>
<th>Estimate</th>
<th>Std.Error</th>
<th>DF</th>
<th>Crit.Value</th>
<th>P.Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>richness</td>
<td>freezing</td>
<td>1.1707</td>
<td>0.5470</td>
<td>531</td>
<td>2.1402</td>
<td>0.0328</td>
</tr>
<tr>
<td></td>
<td>NDVI</td>
<td>-0.0004</td>
<td>0.0002</td>
<td>529</td>
<td>-2.0862</td>
<td>0.0374</td>
</tr>
<tr>
<td></td>
<td>NDVI</td>
<td>-0.0355</td>
<td>0.0023</td>
<td>529</td>
<td>-15.6564</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>NDVI</td>
<td>-4.2701</td>
<td>0.1329</td>
<td>529</td>
<td>-32.1406</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>~freezing</td>
<td>0.2961</td>
<td>NA</td>
<td>531</td>
<td>7.1436</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘ ’ 1
6.4 SEM Example 2. “Independent”
6.4 SEM Example 2. Spatial weights

coordinates(boreal) <- ~x+y

nb <- tri2nb(boreal)

plot(nb, coordinates(boreal))
boreal.sem2 <- psem(

  lagsarlm(richness ~ freezing, data = boreal, listw = nb2listw(nb), tol.solve = 1e-11),

  lagsarlm(NDVI ~ richness + freezing + wet, data = boreal, listw = nb2listw(nb), tol.solve = 1e-11),

  freezing %~~% wet,

  data = boreal)

6.4 SEM Example 2. Spatial weights
6.4 SEM Example 2. Spatial weights

Goodness-of-fit:

   Global model: Fisher's C = 1.774 with P-value = 0.412 and on 2 degrees of freedom

   Individual R-squared:
       Response R.squared
       richness     NA
       NDVI         NA
6.4 SEM Example 2. Spatial weights

Structural Equation Model of boreal.sem2

Call:
  richness ~ freezing
  NDVI ~ richness + freezing + wet
  freezing ~~ wet

  AIC       BIC
  21.774    64.559

Tests of directed separation:

  Independ.Claim Estimate Std.Error DF Crit.Value P.Value
  richness  ~  wet + ... -25.6601  31.2746 NA  -0.8205  0.4119

Coefficients:

  Response  Predictor Estimate Std.Error  DF    Crit.Value P.Value Std.Estimate
  richness   freezing    0.6808   0.5094   NA   1.3364  0.1814       0.0538
   NDVI   richness  -0.0003   0.0001   NA  -1.8619  0.0626    -0.0338
   NDVI  freezing    -0.0207   0.0023   NA  -9.1514  0.0000    -0.2016 ***
   NDVI      wet     -3.1033   0.1519   NA -20.4340  0.0000    -0.5135 ***
~~freezing   ~~wet    0.2961 NA 531    7.1436  0.0000       0.2961 ***

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘ ’ 1
6.4 SEM Example 2. Comparison

freezing

wet

NDVI

richness

freezing

wet

NDVI

richness